Collective Properties and Structure of Heavy and Superheavy Nuclei

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Ground-state collective properties of heaviest nuclei, such as deformations and rotational energies, are studied. In particular, the role of deformations of various multipolarities in the rotational energies is discussed. A large region of even-even nuclei with proton, Z = 82-126, and neutron, N = 126-190, numbers is considered.

1. Introduction

The activity of various groups in studies of heaviest nuclei is fast increasing. Number of institutes performing experimental studies is growing. To laboratories, in which experimental studies are being done for already a long time^{1–4} join new ones.^{5–7} Besides synthesis and investigations of decay modes of these nuclei, also spectroscopic studies of them are performed.^{8–10}

Also theoretical studies intensify. Besides traditional macroscopic-microscopic approaches (e.g. References 11–14), fully microscopic approximations (e.g. References 15–22) are used.

One of the objectives of theoretical research are shell effects. These effects are especially well seen in heaviest nuclei, as these nuclei just exist due to these effects (e.g. Reference 23). Also shapes of these nuclei is a quite interesting question, as these shapes are expected to be quite complex. In particular, deformations of higher multipolarities are expected to play a more important role in these nuclei than in lighter ones. For example, deformations of as high multipolarity as 6 may decrease the energy (i.e. increase the binding energy) of a nucleus by up to about 1.5 MeV.²⁴

The objective of this paper is a short discussion of collective properties of heaviest deformed nuclei. These are such properties as deformations and rotational properties. In particular, the role of deformations of various multipolarities played in rotational properties of these nuclei is discussed.

2. Method of the Calculations

The ground-state energy of a nucleus is calculated in a macroscopic-microscopic approach. The Yukawa-plus-exponential model²⁵ is taken for the macroscopic part of the energy and the Strutinski shell correction is used for the microscopic part. The Woods-Saxon single-particle potential, with the universal variant of its parameters found in Reference 26 and also specified explicitly in Reference 11, is taken as a basis for the shell correction.

Moment of inertia of a nucleus is calculated in the cranking approximation (cf. e.g. Reference 27 and references given therein).

3. Shell Structure

Figure 1 shows a contour map of the ground-state shell correction, $E_{\rm sh}$, calculated for a large region of nuclei with proton number Z = 82-120 and neutron number $N = 126-190.^{28}$ One can see that, besides reproducing minimum of energy for the experimentally well-known doubly magic spherical nucleus 208 Pb, the figure predicts two minima in the superheavy region: one around the nucleus 270 Hs, predicted to be deformed, and the other around the nucleus 298 114, predicted to be spherical. It is interesting to note that the two minima, corresponding to deformed and spherical nuclei, are of comparable depths.



Figure 1. Contour map of the ground-state shell correction energy $E_{\rm sh}$.

Figure 2 illustrates neutron single-particle spectra for the three double magic nuclei, for which the minima of the energy $E_{\rm sh}$ have been obtained in Figure 1. One can see that a large energy gap, obtained in a rather uniform spectrum of doubly de-



Figure 2. Neutron single-particle energy levels calculated for the doubly magic nuclei: ²⁰⁸Pb, ²⁷⁰Hs, and ²⁹⁸114.

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Figure 3. Contour maps of the equilibrium deformations β_1^0 , $\lambda = 2, 4, 6, 8$, plotted as functions of proton Z and neutron N numbers.

generate neutron levels, at the neutron number N = 162 (as well as a smaller gap at N = 152), is a quite spectacular effect.

4. Role of Deformations of Various Multipolarities

The equilibrium deformation parameters β_{λ}^{0} of a nucleus are found by minimization of the potential energy *E* in the deformation space { β_{λ} }. As also higher multipolarity components are found to be important for heaviest nuclei,²⁹ a large, 7-dimensional deformation space { β_{λ} }, $\lambda = 2, 3, ..., 8$, is taken in the analysis.

Figure 3 shows values of β_{λ}^{0} , $\lambda = 2$, 4, 6, 8, obtained in such an analysis.³⁰ Here, only even-multipolarity components β_{λ}^{0} of deformation are shown, as the odd ones β_{λ}^{0} , $\lambda = 3$, 5, 7, are obtained different from zero only for a very few nuclei in the considered region.

One can see in Figure 3 that most of the investigated nuclei are deformed. Only two, relatively small, regions of spherical nuclei appear: one (smaller) region of nuclei near to those with closed neutron shell at N = 126, and the other (larger) near to nuclei with closed neutron shell at N = 184. Relatively small effects of weaker proton spherical closed shells at Z = 82 and Z = 114, on shapes of nuclei, are also visible.

One can also see that the main, quadrupole, component of the deformation, β_2^0 , is the biggest. It is large ($\beta_2^0 \approx 0.24$) and about constant in a large part of the studied region and rapidly decreases as one moves to the boundaries of this region. The higher-multipolarity components change sign as one moves across the region. Although smaller than β_2^0 , these components play a significant role, more significant than for lighter nuclei, as mentioned in the Introduction.

Here, we will illustrate this role in the calculated value of energy of the lowest 2+ state of an even-even nucleus. This state is expected to be of rotational nature. Measurement of this energy, for a nucleus in the region around the nucleus ²⁷⁰Hs, may solve an important problem, if these nuclei are really deformed, as expected on the theoretical ground. The problem has been extensively discussed in Reference 27.

Figure 4 shows the energy E_{2+} calculated for a large region

of even-even heaviest nuclei. One can see that the lowest values of E_{2+} are in the range of 40–55 keV. They are obtained for nuclei with closed neutron shells at N = 152 and 162. When one moves, however, with N off the closed shells, values of E_{2+} fast increase.

It is interesting to learn the dependence of E_{2+} on the dimension λ_{max} of the deformation space, with the use of which the energy E_{2+} is calculated. Figure 5 shows this dependence for four heavy nuclei: ²⁵²No, ²⁵⁴No, ²⁶⁶Sg, and ²⁷⁰Hs. One can see that, for the nucleus ²⁵⁴No, the inclusion of the hexadecapole component β_4 of deformation to the analysis has almost no effect on E_{2+} , while the inclusion of deformation of so high multipolarity as 6, β_6 , is very important. It decreases E_{2+} by about 20%.

For ²⁷⁰Hs, however, the hexadecapole deformation β_4 is crucial. Its inclusion decreases E_{2+} by more than 30%. Example of ²⁶⁶Sg shows that also β_8 may have a significant influence on E_{2+} . Figure 5 tells us how important is the role of deformations of various multipolarities for E_{2+} , and how fast it changes when one moves from one nucleus to another. Thus, although indirectly, measurement of E_{2+} may tell us quite much about the (complex) shape of a very heavy nucleus.



Figure 4. Dependence of the energy E_{2+} on neutron number *N*, calculated for elements with proton number Z = 102-112. For each element, values of considered *N* are specified below the value of *Z*.



Figure 5. Dependence of energy E_{2+} on the dimension λ_{max} of deformation space used in the calculation of this energy.

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